1. A car of mass 800 kg pulls a trailer of mass 200 kg along a straight horizontal road using a light towbar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes 400 N and 200 N respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude 1200 N. Find
(a) the acceleration of the car and trailer,
(b) the magnitude of the tension in the towbar.

The car is moving along the road when the driver sees a hazard ahead. He reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the car of magnitude $F$ newtons and the car and trailer decelerate. Given that the resistances to motion are unchanged and the magnitude of the thrust in the towbar is 100 N ,
(c) find the value of $F$.

## 2.



Two particles $P$ and $Q$, of mass 2 kg and 3 kg respectively, are joined by a light inextensible string. Initially the particles are at rest on a rough horizontal plane with the string taut. A constant force $\mathbf{F}$ of magnitude 30 N is applied to $Q$ in the direction $P Q$, as shown in the diagram above. The force is applied for 3 s and during this time $Q$ travels a distance of 6 m . The coefficient of friction between each particle and the plane is $\mu$. Find
(a) the acceleration of $Q$,
(b) the value of $\mu$,
(c) the tension in the string.
(d) State how in your calculation you have used the information that the string is inextensible.

When the particles have moved for 3 s , the force $\mathbf{F}$ is removed.
(e) Find the time between the instant that the force is removed and the instant that $Q$ comes to rest.
3. A particle $P$ of mass 2 kg is moving under the action of a constant force $\mathbf{F}$ newtons. When $t=0$, $P$ has velocity $(3 \mathbf{i}+2 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$ and at time $t=4 \mathrm{~s}, P$ has velocity $(15 \mathbf{i}-4 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$. Find
(a) the acceleration of $P$ in terms of $\mathbf{i}$ and $\mathbf{j}$,
(b) the magnitude of $\mathbf{F}$,
(c) the velocity of $P$ at time $t=6 \mathrm{~s}$.
4.


A box of mass 30 kg is being pulled along rough horizontal ground at a constant speed using a rope. The rope makes an angle of $20^{\circ}$ with the ground, as shown in the diagram above. The coefficient of friction between the box and the ground is 0.4 . The box is modelled as a particle and the rope as a light, inextensible string. The tension in the rope is $P$ newtons.
(a) Find the value of $P$.

The tension in the rope is now increased to 150 N .
(b) Find the acceleration of the box.
5. A stone $S$ is sliding on ice. The stone is moving along a straight horizontal line $A B C$, where $A B$ $=24 \mathrm{~m}$ and $A C=30 \mathrm{~m}$. The stone is subject to a constant resistance to motion of magnitude 0.3 N. At $A$ the speed of $S$ is $20 \mathrm{~m} \mathrm{~s}^{-1}$, and at $B$ the speed of $S$ is $16 \mathrm{~m} \mathrm{~s}^{-1}$. Calculate
(a) the deceleration of $S$,
(b) the speed of $S$ at $C$.
(c) Show that the mass of $S$ is 0.1 kg .

At $C$, the stone $S$ hits a vertical wall, rebounds from the wall and then slides back along the line $C A$. The magnitude of the impulse of the wall on $S$ is 2.4 Ns and the stone continues to move against a constant resistance of 0.3 N .
(d) Calculate the time between the instant that $S$ rebounds from the wall and the instant that $S$ comes to rest.
6.


A sledge has mass 30 kg . The sledge is pulled in a straight line along horizontal ground by means of a rope. The rope makes an angle $20^{\circ}$ with the horizontal, as shown in the diagram above. The coefficient of friction between the sledge and the ground is 0.2 . The sledge is modelled as a particle and the rope as a light inextensible string. The tension in the rope is 150 N. Find, to 3 significant figures,
(a) the normal reaction of the ground on the sledge,
(b) the acceleration of the sledge.

When the sledge is moving at $12 \mathrm{~m} \mathrm{~s}^{-1}$, the rope is released from the sledge.
(c) Find, to 3 significant figures, the distance travelled by the sledge from the moment when the rope is released to the moment when the sledge comes to rest.
7. A particle $P$ of mass 0.4 kg is moving under the action of a constant force $\mathbf{F}$ newtons. Initially the velocity of $P$ is $(6 \mathbf{i}-27 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$ and 4 s later the velocity of $P$ is $(-14 \mathbf{i}+21 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$.
(a) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the acceleration of $P$.
(b) Calculate the magnitude of $\mathbf{F}$.

1. (a) For whole system: $1200-400-200=1000 a$

$$
a=0.6 \mathrm{~m} \mathrm{~s}^{-2}
$$

A1 3
(b) For trailer: $T-200=200 \times 0.6$

$$
T=320 \mathrm{~N}
$$

A1 ft
A1

## OR:

A1 ft
A1 3
(c) For trailer: $200+100=200 f$ or $-200 f$

$$
f=1.5 \mathrm{~m} \mathrm{~s}^{-2}(-1.5)
$$

For car: $400+F-100=800 f$ or $-800 f$

$$
F=900
$$

(N.B. For both: $400+200+F=1000 f$ )
2. (a)

$s=u t+\frac{1}{2} a t^{2} \Rightarrow 6=\frac{1}{2} a \times 9$
$a=1 \frac{1}{3}\left(\mathrm{~m} \mathrm{~s}^{-2}\right)$
A1 2
(b) N2L for system $30-\mu 5 g=5 a$
ft their $a$, accept symbol M1A1ft
$\mu=\frac{14}{3 g}=\frac{10}{21}$ or awrt 0.48
DM1A1 4
(c) N2L for $P T-\mu 2 g=2 a$
$T-\frac{14}{3 g} \times 2 g=2 \times \frac{4}{3}$
Leading to $T=12$ (n)
awrt 12
DM1A1
4
Alternatively
N2L for $Q$
$30-T-\mu 3 g=3 a$
M1A1
Leading to $T=12$ (n)
awrt 12
DM1A1
(d) The acceleration of $P$ and $Q$ (or the whole of the system) is the same.
(e) $\quad v=u+a t \Rightarrow v=\frac{4}{3} \times 3=4$

B1ft on $a$
N2L (for system or either particle)
$-5 \mu g=5 a$
or equivalent
$a=-\mu g$
$v=u+a t \Rightarrow 0=4-\mu g t$
DM1
Leading to $t=\frac{6}{7}$ (s)
accept $0.86,0.857$
A1 4
3. (a) $\mathbf{a}=\frac{(15 \mathbf{i}-4 \mathbf{j})-(3 \mathbf{i}+2 \mathbf{j})}{4}=3 \mathbf{i}-15 \mathbf{j}$
(b) N2L $\mathbf{F}=m \mathbf{a}=6 \mathbf{i}-3 \mathbf{j}$
$\begin{aligned} \text { ft their a } & \text { M1A1 } \\ \text { accept } \sqrt{ } 45 \text {, awrt } 6.7 & \text { M1A1 }\end{aligned}$
$|\mathbf{F}|=\sqrt{ }\left(6^{2}+3^{2}\right) \approx 6.71(\mathrm{~N})$
accept $\sqrt{ } 45$, awrt $6.7 \quad$ M1A1 4
(c) $\quad \mathbf{v}_{6}=(3 \mathbf{i}+2 \mathbf{j})+(3 \mathbf{i}-1.5 \mathbf{j}) 6$

$$
=21 \mathbf{i}-7 \mathbf{j}\left(\mathrm{~m} \mathrm{~s}^{-1}\right)
$$

ft their a M1A1ft
A1 1
4. (a)
$\mu R$


Use of $F=\mu R$
B1
$\Phi P \cos 20^{\circ}=\mu R$
M1A1
i $R+P \sin 20^{\circ}=30 g$
A1
$P \cos 20^{\circ}=\mu\left(30 g-P \sin 20^{\circ}\right)$
$P=\frac{0.4 \times 30 g}{\cos 20^{\circ}+0.4 \sin 20^{\circ}}$
$\approx 110(\mathrm{~N})$
accept 109

A1 8

5. (a) $16^{2}=20^{2}-2 \times a \times 24 \Rightarrow a=\underline{3 \mathrm{~m} \mathrm{~s}^{-2}}$

A1 2
(b) $v^{2}=20^{2}-2 \times 3 \times 30$ A1ft
$v=\sqrt{ } 220$ or $14.8 \mathrm{~m} \mathrm{~s}^{-1}$
A1 3
(c) $0.3=m \times 3 \Rightarrow m=0.1 \mathrm{~kg}\left({ }^{*}\right)$

A1 2
(d) $0.1(w+\sqrt{220})=2.4$
$w=9.17$
$0=9,17-3 \times t$
$t \approx \underline{3.06 \mathrm{~s}}$
A1 6
6. (a)


$$
\mathrm{R}(\uparrow) R+150 \sin 20=30 g
$$

A1
$\Rightarrow R \approx \underline{243 \mathrm{~N}}$
A1 3
(b) $\mathrm{R}(\rightarrow): 150 \cos 20-0.2 R=30 a$ A1
$\Rightarrow a \approx \underline{3.08 \mathrm{~m} \mathrm{~s}^{-2}}$
A1 3
(c)


$$
\begin{array}{ll}
S=30 g \Rightarrow F=0.2 \times 30 g & \text { A1 } \\
30 a^{\prime}=(-) 0.2 \times 30 g \Rightarrow a \\
0=12^{2}-2 \times 0.2 g \times s(-) 0.2 g(=1.96) & \text { A1 } \\
\Rightarrow s \approx 36.7 \mathrm{~m} & \\
\left.\hline \underline{3} \text { (using new } a^{\prime}\right) & \text { A1 }
\end{array}
$$

7. (a) $\mathbf{a}=[-14 \mathbf{i}+21 \mathbf{j}-(6 \mathbf{i}-27 \mathbf{j})] \div 4$ A1
$=(-5 \mathbf{i}+12 \mathbf{j}) \mathrm{m} \mathrm{s}^{-2}$
(b) $\quad|\mathbf{a}|=\sqrt{ }\left(5^{2}+12^{2}\right)=13$

$$
|\mathbf{F}|=m|\mathbf{a}|=0.4 \times 13=5.2 \mathrm{~N}
$$

A1 3

Alt (b)
$\mathbf{F}=0.4(5 \mathbf{i}+12 \mathbf{j})=2 \mathbf{i}+4.8 \mathbf{j}$
$|\mathbf{F}|=\sqrt{ }\left(2^{2}+4.8^{2}\right)=5.2 \mathrm{~N}$
A1 3

1. Part (a) was well done by the majority of candidates and a good number went on to use the answer correctly in part (b). If mistakes were made they were the usual sign errors or more seriously, in terms of marks lost, missing terms.
The third part was poorly done. There was confusion over the direction of the forces and the concept of thrust. A few candidates halved the thrust and used 50 N in each equation. Some used the values of the acceleration and tension from previous parts.
2. Full marks were rarely achieved in this question. Some made a poor start by using $F=m a$ in part (a) rather than an appropriate constant acceleration formula. In the second part many used separate equations of motion for the two particles (sometimes with extra or omitted terms) but then not uncommonly solved them as simultaneous equations with the same $F$ (friction term), showing a lack of understanding of the problem. Only a minority used the more straightforward 'whole system' approach. There was some recovery in part (c) where follow through marks were available as long as the 'appropriate' terms were included in the equation of motion of one particle. A significant number of candidates knew that an inextensible string implied that the accelerations of the two particles were the same in part (d), but some of those went on to incorrectly mention the tension as well and so lost the mark. Many candidates who reached part (e) seemed to know they had to find the new deceleration but lost marks by including a tension or the 30 N in their equation of motion.
3. In part (a) most candidates knew the method and it was often fully correct but a number failed to find the magnitude of the force in the second part, with some, subtracting the squares of the components instead of adding them. Part (c) was well answered.
4. Good candidates found this question reasonably straightforward, but many of the weaker ones lost significant numbers of marks because they thought that $\mathrm{R}=30 \mathrm{~g}$. It was odd that many candidates could get part (a) completely correct but then were unable to make any progress at all with part (b) and didn't appreciate the similarity between them. Some marks were again lost due to over-accurate answers. A clearly labelled diagram in each part made a huge difference.
5. Most could make good attempts at the first three parts of the question, though a misreading of the information (confusing ' $A C$ ' and ' $B C$ ' was not uncommon). In part (d) the most common mistake was to confuse signs again (similar to qu.1) in writing down the impulse-momentum equation, but most could then go on to use their result in an appropriate way to get a value for the time.
6. Parts (a) and (b) were well done by some with few writing down that the normal reaction in (a) was equal to the weight; in part (b) several missed out a force in trying to find the acceleration However, many failed to observe the specific demand to give their answers to 3 significant figures. In part (c), many failed to realise that the normal reaction would change: hence they
used their working from the earlier parts of the question to find the frictional force; others used the answer form part (b) (for acceleration) in their working for part (c) (where the sledge is decelerating!).
7. This was generally very well done. A few errors occurred in part (a) with candidates using $u-v$, instead of $v-u$, and there were some arithmetical slips. In part (b), a few simply found the magnitude of their acceleration, failing to multiply by the mass; quite a few simply found the force as a vector, failing to understand the significance of the word 'magnitude' here.
